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Perturbative analysis of the $2\nu\beta\beta$ decays of ^{100}Mo and ^{116}Cd

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Abstract. We have performed a theoretical analysis of the ground-state–to–ground-state transitions in ^{100}Mo and ^{116}Cd, based on the quasiparticle random-phase approximation and on a straightforward perturbative scheme. The results show that the single-state dominance found in the realistic calculations of the nuclear matrix elements, which is consistent with data, can be viewed as a result of the interference between few two-quasiparticle configurations.

PACS. 23.40.Bw Weak-interaction and lepton (including neutrino) aspects – 23.40.Hc Relation with nuclear matrix elements and nuclear structure – 21.60.Jz Hartree-Fock and random-phase approximations – 27.60.+j $90 \leq A \leq 149$

1 Introduction

Nuclear double-beta-decay transitions have been intensively studied theoretically as well as experimentally, as given in recent reviews and references therein [1–7]. The half-lives of the already observed two-neutrino mode of double-beta decay ($2\nu\beta\beta$) are the longest ones ever measured. In addition, very stringent half-life limits for the neutrinoless mode have been obtained [3–12]. Generally speaking, the complete theoretical understanding of these transitions is still a challenging question with obvious consequences upon the test of both electroweak interactions and nuclear structure. Due to the perturbative nature of the problem, in the weak-interaction sector of the theory, the calculation of nuclear matrix elements of two-neutrino double-beta-decay transitions involves a summation over the 1^+ states of the intermediate nucleus participant in the decay chain connecting the initial and final even-even mass nuclei [1–5]. Thus two single-beta-decay transitions are involved, connecting the initial and final ground states to the ground and excited states of the participant intermediate odd-odd nucleus. These virtual transitions can be investigated experimentally by charge-exchange reactions on the initial and final nuclei [4] or by induced electron-capture and β-decays from the double-odd-mass nucleus.

Considering these possibilities, it has been suggested [13] that, for those $2\nu\beta\beta$ transitions where the ground state of the intermediate nucleus is a $J^\pi = 1^+$ state, the transition matrix element could be governed by two virtual transitions, the first one going from the initial ground state to the 1^+ ground state of the intermediate nucleus and the second one going from this 1^+ state to the final ground state. This assumption is known as the *Single-State-Dominance* (SSD) and its validity has been studied experimentally and theoretically in [14–17]. Data on the SSD have been reported by García *et al.* [15], by Akimune *et al.* [16] and by Bhattacharya *et al.* [17]. The dominance of low-lying single particle-hole states in intermediate nuclei have been shown experimentally for $2\nu\beta\beta$ decays in medium-mass nuclei and it was analyzed in terms of the couplings to GT giant resonances by Ejiri and Toki in [14]. A recent theoretical discussion of the SSD was presented in [18]. The SSD has been tested in EC measurements at Notre Dame [17] and in $(^3\mathrm{He}, t)$ experiments by the Osaka group [16].

The influence of the high-lying energy states, as the GT giant resonance, on the matrix elements of the $2\nu\beta\beta$ decay channels has been studied by M. Ericson, T. Ericson and P. Vogel [19]. In their paper the authors have advanced qualitatively the notion that low-lying 1^+ states dominate the decay. This finding was confirmed by the results of realistic calculations, although the connection between them and the schematic analysis of [19] has not been studied in detail.

In this article we show the results of calculations of the two-neutrino double-beta decays of ^{100}Mo and

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^{116}Cd, since the $2\nu\beta\beta$ experimental data are available for them [8–11], and they are among the systems where the contributions to the decay rates coming from low-lying intermediate states can be verified. The results can, in fact, be tested both theoretically and experimentally. Our theoretical analysis was performed in the complete QRPA framework and also in a perturbative framework based on dominant components of the wave functions. The study has been motivated by the need to understand, and eventually to predict, the conditions under which the coherent contributions of low-lying 1^+ states and the SSD mechanism can occur. Within this context a better estimate of nuclear matrix elements relevant for $2\nu\beta\beta$ may be achieved. This becomes an important issue concerning the planning of next-generation double-beta-decay experiments.

A brief description of the formalism is presented in sect. 2. Results and discussions are given in sect. 3. Conclusions are drawn in sect. 4.

2 Formalism

For the benefit of the reader we are presenting in the next subsection the essentials of the formalism currently used to calculate the spectrum of double-odd-mass nuclei and the transition rates of $2\nu\beta\beta$ decay processes. Although the formulae are well known and have been published somewhere else, they are needed here in order to establish the link between the full QRPA calculation and the perturbative analysis. The correponding formalisms are given in subsects. 2.1 and 2.2, respectively.

2.1 Nuclear matrix elements in the QRPA method

Following the notation of [5] the proton-neutron excitations (pn-excitations) of a nucleus with open shells are described by the two-quasiparticle creation and annihilation operators

$$A^\dagger(pn,JM) = [\alpha_p^\dagger \alpha_n^\dagger]_{JM}\,;$$
$$\tilde{A}(pn,JM) = (-1)^{J+M}\left(A^\dagger(pn,J,-M)\right)^\dagger\,. \quad (1)$$

The linear combination of them defines the one-phonon creation operator

$$Q^\dagger_{JM}(m) = \sum\nolimits_{pn}\Big[X_{pn}(J^\pi,m)A^\dagger(pn,JM) - Y_{pn}(J^\pi,m)\tilde{A}(pn,JM)\Big]\,, \quad (2)$$

which is acting on the correlated QRPA vacuum. The annihilation operator is defined similarly. The treatment of pn-excitations in the QRPA basis was first introduced by Halbleib and Sorensen [20]. A brief outline of the formalism is given below.

The pn-QRPA equations have the general form

$$\begin{pmatrix} A & B \\ B & A \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} = \Omega \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix}\,, \quad (3)$$

where the metric matrix on the right-hand side has a simple diagonal form. In this model the states representing the excited states of a odd-odd nucleus are given by a diagonalization procedure where the corresponding amplitudes are determined from the non-Hermitian eigenvalue problem of eq. (3). In the basic form of the pn-QRPA the sub-matrices A and B are given by the equations

$$A_{pn,p'n'} = \delta_{pp'}\delta_{nn'}(E_p+E_n) - 2g_{\rm pp}G(pnp'n',J)(u_pu_nu_{p'}u_{n'} + v_pv_nv_{p'}v_{n'}) - 2g_{\rm ph}F(pnp'n',J)(u_pv_nu_{p'}v_{n'} + v_pu_nv_{p'}u_{n'})\,, \quad (4)$$

$$B_{pn,p'n'} = 2g_{\rm pp}G(pnp'n',J)(u_pu_nv_{p'}v_{n'} + v_pv_nu_{p'}u_{n'}) - 2g_{\rm ph}F(pnp'n',J)(u_pv_nv_{p'}u_{n'} + v_pu_nu_{p'}v_{n'})\,, \quad (5)$$

where E_p and E_n are the proton and neutron quasiparticle energies. The quantities $G(pnp'n',J)$ and $F(pnp'n',J)$ are the particle-particle and particle-hole two-body matrix elements of the residual proton-neutron interactions, as defined by Baranger in [21]. The u and v factors appearing in (4) and (5) are the usual BCS occupation factors. The coefficients $g_{\rm ph}$ and $g_{\rm pp}$ are overall scaling factors of the two-body matrix elements in the particle-hole and particle-particle channels, respectively. The parameter $g_{\rm ph}$ is determined by requiring that data on the excitation energy of the Gamow-Teller giant resonance is reproduced by the pn-QRPA solutions. Here we fix the factor $g_{\rm pp}$ by data on single beta-decays.

The importance of the terms proportional to $g_{\rm pp}$ in the pn-QRPA matrix, in relation with the suppression of the matrix elements of the two-neutrino double-beta-decay mode, was pointed out by J. Engel, P. Vogel and M. Zirnbauer [22] and by O. Civitarese, A. Faessler and T. Tomoda [23]. The authors of [22] based their conclusions on the results obtained by using a schematic proton-neutron interaction, while in [23] a realistic two-body interaction was utilized. In both cases the renormalization of the so-called particle-particle channels was found to induce the suppression of the total matrix element, relevant for the two-neutrino double-beta decay, as we shall also discuss later on in this work.

By adopting the standard BCS transformations between the single-particle and quasiparticle states, one has for the two-quasiparticle terms of the one-body beta-decay operators the structure

$$M^-_{\lambda\mu} = \sqrt{\tfrac{1}{3}}\sum_{pn}(p\parallel M_\lambda\parallel n) \times[u_pv_nA^\dagger(pn,\lambda\mu) + v_pu_n\tilde{A}(pn,\lambda\mu)]\,,$$
$$M^+_{\lambda\mu} = -\sqrt{\tfrac{1}{3}}\sum_{pn}(p\parallel M_\lambda\parallel n) \times[v_pu_nA^\dagger(pn,\lambda\mu) + u_pv_n\tilde{A}(pn,\lambda\mu)]\,,$$

where M_λ is the $M^\pm_\lambda$ operator without the isospin ladder operators $\tau^\pm$, and the quasiparticle-pair operators A and $\tilde{A}$ have been defined before. The QRPA expressions for

the β^-- and β^+-decay amplitudes are

$$(J^\pi, m \parallel M^-_\lambda \parallel \mathrm{QRPA}) = \delta_{J\lambda} \sum_{pn} (p \parallel M_\lambda \parallel n)[u_p v_n X_{pn}(J^\pi, m) + v_p u_n Y_{pn}(J^\pi, m)], \quad (6)$$

$$(J^\pi, m \parallel M^+_\lambda \parallel \mathrm{QRPA}) = -\delta_{J\lambda} \sum_{pn} (p \parallel M_\lambda \parallel n)[u_n v_p X_{pn}(J^\pi, m) + u_p v_n Y_{pn}(J^\pi, m)], \quad (7)$$

where $|\mathrm{QRPA}\rangle$ is the correlated ground state of the QRPA.

For the case of the two-neutrino double-beta-decay mode the transitions are mediated by the allowed GT operators $M^\pm_{1\mu} = \sum_j \sigma_\mu(j)\tau^\pm(j)$ and are described as second-order processes in the weak interactions. They involve virtual excitations with $J^\pi = 1^+$ of the initial and final ground states. Final transition rates are obtained by integrating over leptonic variables and by performing the sum over intermediate virtual nuclear excitations [5]. The inverse half-life for a two-neutrino double-beta-decay transition from the initial ground state 0^+_i to the final ground state 0^+_f reads

$$[t^{(2\nu)}_{1/2}(0^+_i \to 0^+_f)]^{-1} = G^{(2\nu)}_{\mathrm{DGT}} \mid M^{(2\nu)}_{\mathrm{DGT}} \mid^2 , \quad (8)$$

where $G^{(2\nu)}_{\mathrm{DGT}}$ is the integral over the phase space of the leptonic variables [2,5]. The nuclear matrix element $M^{(2\nu)}_{\mathrm{DGT}}$ can be written as

$$M^{(2\nu)}_{\mathrm{DGT}} = \frac{\sum_{m,n} (0^+_f \mid\mid M^-_1 \mid\mid 1^+_m) \langle 1^+_m \mid 1^+_n \rangle (1^+_n \mid\mid M^-_1 \mid\mid 0^+_i)}{(\frac{1}{2}Q_{\beta\beta} + E_m - M_i)/m_e + 1}. \quad (9)$$

The overlap $\langle 1^+_m \mid 1^+_n \rangle$ between the two sets of 1^+ states, which are pn-QRPA solutions based on the initial and final ground states, is included to match the two branches of virtual excitations. Here we have not included contributions of Fermi transitions through the isobaric analog states (IAS), since the transition from the IAS to the final nucleus is negligible because of the isospin symmetry.

2.2 Perturbative approach

In order to grasp the physical mechanism leading to the actual calculated values of the nuclear matrix elements $M^{(2\nu)}_{\mathrm{DGT}}$ we have performed a perturbative analysis. Intermediate $J^\pi = 1^+$states concerned are two quasi-particle states in the low-excitation region of the spectrum and the GT giant resonance (GTGR) in the high-excitation region. The GTGR, which absorbs a large fraction of the single-β-decay strength, has been found to have little contribution to the $2\nu\beta\beta$. This is consistent with the results of the qualitative analysis advanced by M. Ericson *et al.* in [19]. Thus we consider only the low-lying two-quasiparticle states. The virtual excitations consist of quasiproton-quasineutron pairs coupled to $J^\pi = 1^+$. Only pairs where the quasiproton and the quasineutron have the same value of the orbital quantum number l contribute to the virtual transitions. Different pairs may have different l values. We now concentrate on the situation where two low-energy pairs are active and let them mix by diagonalizing the residual interaction in a two-by-two perturbative scheme. We can distinguish between two possibilities for the individual orbitals entering in the pairs, namely: 1) the active particles are occupying spin-orbit doublets, so that all pairs have the same value of l, or 2) one can think of single-particle states with different values of l, above ($l_>$) and below ($l_<$) the Fermi surface.

Case 1: neutrons and protons in only one l-orbital

To begin with we shall assume that the valence space is spanned by spin-orbit-partner orbitals $j_+ = l + 1/2$ and $j_- = l - 1/2$, both for protons (p) and neutrons (n). We shall construct the proton-neutron quasiparticle basis by including the following dominant configurations:

$$\mid 1\rangle = \mid j_+(p) j_-(n)\rangle; \qquad \mid 2\rangle = \mid j_-(p) j_-(n)\rangle, \quad (10)$$

with unperturbed energies

$$E_1 = \epsilon_+(p) + \epsilon_-(n); \qquad E_2 = \epsilon_-(p) + \epsilon_-(n) , \quad (11)$$

respectively, where ϵ are quasiparticle energies. We shall represent the perturbed states by the linear combinations

$$\mid I\rangle = \alpha \mid 1\rangle + \beta \mid 2\rangle, \quad \mid II\rangle = \alpha \mid 2\rangle - \beta \mid 1\rangle \quad (12)$$

and these are the solutions of the eigenvalue equation corresponding to the Hamiltonian matrix $M_{ij} = E_i\delta_{ij} + H_{ij}$. The interaction matrix elements are defined by $H_{ij} = \langle i \mid V \mid j\rangle$. For the present calculations we shall use the effective pair energies $\tilde{E}_i = E_i + H_{ii}$. The non-diagonal terms $H_{12} = H_{21} = h$ are assumed to be smaller than the energy difference, $e = \tilde{E}_2 - \tilde{E}_1$, between the effective energies. Under these assumptions we can write the perturbed eigenvalues approximately as

$$\lambda_+ = \tilde{E}_2 + \delta^2 e; \quad \lambda_- = \tilde{E}_1 - \delta^2 e \quad , \quad (13)$$

where $\delta = h/e$. The corresponding approximate eigenvectors read

$$\mid +\rangle = \eta \mid 2\rangle + \delta \mid 1\rangle; \qquad \mid -\rangle = -\eta \mid 1\rangle + \delta \mid 2\rangle, \quad (14)$$

where $\eta = \sqrt{1 - \delta^2}$. At this point we are in conditions to calculate a double-beta-decay transition from the initial ground state $\mid i\rangle$ to the final ground state $\mid f\rangle$. In this scheme the approximate eigenvalues $\lambda_\pm$ are measured relative to the initial ground state, which is taken as the quasiparticle vacuum. The Gamow-Teller operator $\beta^\pm$ connects the initial and final ground states with the states $\mid +\rangle$ and $\mid -\rangle$. The corresponding matrix elements are then given

by

$$\langle + \mid \beta^- \mid i\rangle = \eta\langle 2 \mid \beta^- \mid i\rangle + \delta\langle 1 \mid \beta^- \mid i\rangle,$$
$$\langle - \mid \beta^- \mid i\rangle = -\eta\langle 1 \mid \beta^- \mid i\rangle + \delta\langle 2 \mid \beta^- \mid i\rangle,$$
$$\langle + \mid \beta^+ \mid f\rangle = \eta\langle 2 \mid \beta^+ \mid f\rangle + \delta\langle 1 \mid \beta^+ \mid f\rangle,$$
$$\langle - \mid \beta^+ \mid f\rangle = -\eta\langle 1 \mid \beta^+ \mid f\rangle + \delta\langle 2 \mid \beta^+ \mid f\rangle. \quad (15)$$

The matrix elements representing the two-neutrino double-beta-decay mode is written as in eq. (9), but the complete set of intermediate virtual excitations is restricted to the states $\mid +\rangle$ and $\mid -\rangle$. By inserting into eq. (9) the values of the transition matrix elements given in eq. (15) one obtains the result

$$M_{\rm DGT}^{(2\nu)} = \tilde{F}_1 I_1 \left[\frac{\eta^2}{\lambda_- + T_f} + \frac{\delta^2}{\lambda_+ + T_f}\right] + \tilde{F}_2 I_2 \left[\frac{\delta^2}{\lambda_- + T_f} + \frac{\eta^2}{\lambda_+ + T_f}\right] + (\tilde{F}_1 I_2 + \tilde{F}_2 I_1)\eta\delta \left[\frac{1}{\lambda_+ + T_f} - \frac{1}{\lambda_- + T_f}\right], \quad (16)$$

where in short-hand notation $\tilde{F}_k = \langle f \mid \beta^- \mid k\rangle$, $I_k = \langle k \mid \beta^- \mid i\rangle$ and $T_f = \frac{1}{2}Q_{\beta\beta} + m_e c^2$.

Case 2: neutrons and protons in two different l-orbitals

For this case we shall represent the basis states as

$$\mid 1\rangle = \mid j_<(p) j_<(n)\rangle; \quad \mid 2\rangle = \mid j_>(p) j_>(n)\rangle. \quad (17)$$

In the first configuration the orbitals have $l = l_<$ and in the second one $l = l_>$. The unperturbed energies are

$$E_1 = \epsilon_<(p) + \epsilon_<(n); \quad E_2 = \epsilon_>(p) + \epsilon_>(n)\ , \quad (18)$$

respectively. As done in the previous subsection, one can construct the perturbed eigenfunctions by diagonalizing a two-by-two matrix, where the off-diagonal elements will now represent the interaction between pairs with different values of l. The final expressions will, of course, be the same.

3 Results and discussion

In the following we shall present and discuss the results corresponding to the two-neutrino double-beta decays of ^{100}Mo and ^{116}Cd. For the single-particle basis we have used the solutions of the Woods-Saxon central potential for each mass region included in the calculations. Coulomb corrections were added to describe proton states. The $N = 3, 4$ and 5 major oscillator shells for neutrons and the $N = 3$ and 4 for protons were used. To each shell we have added the intruding orbital. Matrix elements for the effective two-body interaction were extracted from the G-matrix constructed with the Bonn one-boson-exchange potential (OBEP) of [24]. Pairing effects were accounted for by the monopole terms of the interaction, which were renormalized to reproduce the observed mass differences. Single-particle levels around the proton and/or the neutron Fermi surfaces were adjusted as done in ref. [18]. As said before, the otherwise free value of ($g_{\rm pp}$) is fixed by optimizing the theoretical results for the experimentally known single-beta-decay rates for the ground states of the initial and final nuclei and for both cases, ^{100}Mo and ^{116}Cd, the resulting value is $g_{\rm pp} = 1.00$. Details of the calculated spectra corresponding to these decay systems are shown in ref. [18]. From the results shown in this reference (see figs. 6 and 10 of [18]) we clearly see that the SSD is realized in the ground-state–to–ground-state decays of ^{100}Mo and ^{116}Cd.

Table 1. Calculated virtual decay paths in the double-beta decays of ^{100}Mo and ^{116}Cd. For each case we are listing the energy denominator, E_d, of eq. (9), and the matrix elements for the virtual β^- transitions, M_l and M_r, which connect the initial and final ground states with the intermediate $1^+_{\rm g.s.}$ and 1^+_1 states of the double-odd-mass nucleus. Notice that the quantity $E_d = (\frac{1}{2}Q_{\beta\beta} + E_m - M_i)/m_e + 1$ is dimensionless. The last column shows the contribution of each decay path to the final matrix element of eq. (9).

^{100}Mo					
State	E_d	M_l	M_r	$M_l \times M_r$	Contribution
$1^+_{\rm g.s.}$	3.297	−1.87	−0.89	1.664	0.504
1^+_1	12.947	−0.881	0.449	−0.366	−0.028
^{116}Cd					
State	E_d	M_l	M_r	$M_l \times M_r$	Contribution
$1^+_{\rm g.s.}$	3.669	1.360	0.324	0.435	0.118
1^+_1	6.824	1.270	0.003	0.004	0.001

In the work of ref. [18] we have classified the $2\nu\beta\beta$ decays as belonging to one of the two categories: i) decays with clear dominance of the virtual transition going through the 1^+ ground state ($1^+_{\rm g.s.}$) of the odd-odd intermediate nuclei and ii) decays where the total matrix element is of the order of the extracted SSD matrix element, due to interference between transitions through low-lying and high-lying intermediate states. The pn-QRPA results show that the decays of ^{100}Mo and ^{116}Cd belong to the first category. However, because of the different microscopic structure of the $1^+_{\rm g.s.}$ wave functions, in ^{100}Tc and in ^{116}In, the composition of these matrix elements varies from case to case [25].

The relative contributions to each of the transitions, going through the $1^+_{\rm g.s.}$ and the 1^+_1, are shown in table 1. These results are taken from the complete pn-QRPA calculations. The low-energy part of the 1^+ spectrum of ^{100}Tc is represented by $1^+_{\rm g.s.}$ and three states at 4.931 MeV, 5.370 MeV and 5.899 MeV, respectively. In the case of ^{116}I the complete pn-QRPA calculations yield the 1^+ ground state and two low-lying 1^+ states at 1.612 MeV and 4.125 MeV, respectively. These energies, for the two first states, translate into the scaled, dimensionless,

denominators E_d of eq. (9), which are shown in table 1. Some interesting features become evident from the results shown in this table. For the case of ^{100}Mo the virtual transitions have individual matrix elements which are comparable, within factors of the order of two. The product of the single-beta-decay transitions, for the $1^+_{\rm g.s.}$ and for the 1^+_1, are also comparable, but the final contributions are very much governed by the energy denominators. At the end, the transition matrix element which involves the $1^+_{\rm g.s.}$ is 20 times larger than the one corresponding to the 1^+_1 state. We can characterize this decay by noticing that all virtual decay branches are relatively strong but that the energy difference between the 1^+_1 and the $1^+_{\rm g.s.}$ suppresses the interference between these states, leading to a SSD scenario. The case of the decay of ^{116}Cd is somewhat different. There, the virtual transitions contribute with different matrix elements. The energy difference between the 1^+_1 and the $1^+_{\rm g.s.}$ is not so large, as it is in the case of the decay of ^{100}Mo, but because of the suppression of the β^- virtual transition from the 1^+_1 state, due to particle-particle interactions, the relative contributions to the final matrix element differ by a factor of one hundred. Thus, the SSD is fulfilled but for a different reason than in the case of ^{100}Mo.

In order to identify the mechanism responsible for the SSD we have taken the active orbitals and performed the perturbative analysis outlined in subsect. 2.2. The 1^+ ground state of ^{100}Tc is dominated by a single configuration, where the quasiproton and quasineutron occupy the orbits $g_{9/2}$ (proton) and $g_{7/2}$ (neutron)[1]. This is a situation where both the quasiproton and the quasineutron are in orbitals with the same l-value. This case is then a particular limit of the configuration scheme described in subsect. 2.2, case 1). The perturbative expansion is then governed by a single transition, of spin flip character, which yields a transition matrix element very close to the final pn-QRPA one. That is to say that the SSD is just the result of the contribution of a single proton-neutron pair. The pn-QRPA result and the perturbative result for this transition are consistent with the value of $M^{(2\nu)}_{\rm DGT}$ given by that single transition. The next excited 1^+ contributes with less than 6% to the final matrix element[2].

The situation in the decay of ^{116}Cd, on the contrary, seems to be more like the case 2 of subsect. 2.2. The wave function of the 1^+ ground state of ^{116}In has basically two components: $\mid 0g_{9/2}(p)0g_{7/2}(n)\rangle$ pair as the dominant component and the $\mid 1d_{5/2}(p)1d_{3/2}(n)\rangle$ pair as the small component. The wave function of the first excited 1^+ state in ^{116}In has the inverse composition. Because of the selection rules of the allowed GT operator, the virtual transitions can take place between the d-orbitals and g-orbitals separately. Then, both contributions are adding to a matrix element of the order of 0.12, which is also the value of the final matrix element. This situation is clearly realized in the perturbative expansion. There, the two active 1^+ excitations are 2.6 MeV apart and the mixing between them is induced by an off diagonal matrix element of the order of 0.6 MeV. However, the two branches of virtual transitions, from the β^- and β^+ sides of the decay, show some important differences. Both β^- decays are contributing with approximately the same strength while the β^+ branch of the second 1^+ state is very much suppressed. At the end, the result of this cancellation shows up and the final matrix element is quite close to the SSD value.

4 Conclusions

We have analyzed the structure of the pn-QRPA results, for the matrix elements of the ground-state–to–ground-state two-neutrino double-beta decays of ^{100}Mo and ^{116}Cd, in the framework of a simple perturbative approach. We have taken these two cases because in both of them the dominance of the low-lying intermediate states, and thus the presence of the SSD mechanism, has been confirmed experimentally. The comparison between the microscopic (pn-QRPA) and perturbative results suggests that the case of the decay of ^{100}Mo is different from the one of ^{116}Cd. In spite of the presence of relatively close-lying 1^+ excitations, as it is the case of ^{116}In, the nuclear matrix elements of the two-neutrino double-beta-decay mode show clear dominance of the transition going through the 1^+ ground state, without significant fragmentation. The decay of ^{100}Mo can be described as the result of the SSD mechanism although two virtual decay chains can participate. These virtual transitions in ^{100}Mo have comparable yield but one of them, the one going through the first excited 1^+ state of ^{100}Tc, is suppressed by a large energy denominator. The decay of ^{116}Cd is also described by the SSD but this mechanism shows up for a different reason. There, one of the virtual transitions is very weak and the final matrix element is governed by the transitions going through the ground state of ^{116}In, in spite of the fact that the energy of the first excited 1^+ state, as predicted by the pn-QRPA, is much lower than in the case of the excitations in ^{100}Tc. In a quantitative way these results confirm the qualitative analysis of M. Ericson *et al.* [19] and shed new light on the understanding of the pn-QRPA calculations of $2\nu\beta\beta$ decay rates in medium-mass nuclei, like ^{100}Mo and ^{116}Cd, where giant resonances and high-lying complex states are of little importance.

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[1] The actual composition of the QRPA wave function is the following (only the configurations with $\mid X_{pn} \mid > 0.1$ are shown): $1.11 \mid 0g_{9/2}(\pi)0g_{7/2}(\nu)\rangle + 0.135 \mid 1d_{5/2}(\pi)1d_{3/2}(\nu)\rangle$.

[2] The numbers given in the fifth column of table 1 of ref. [18], for the case of ^{100}Mo, should be 0.50 and 0.40 (cases B and C), respectively. The corresponding half-lives are 1.1×10^{18} years and 1.7×10^{18} years.

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